

- (ii) Compute the inverse of the following matrix by using elementary transformation :

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

7. Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

8. (i) Find the area of triangle whose vertices are A (1, -1, 0), B (2, 1, -1) and C (-1, 1, 2).

- (ii) If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$; $\vec{b} = 3\hat{i} - 4\hat{k}$, $\vec{c} = -\hat{i} + \hat{j}$, $\vec{d} = 2\hat{i} - \hat{j} + 3\hat{k}$, then find $\left(\vec{a} \times \vec{b} \right) \times \left(\vec{c} \times \vec{d} \right)$.

9. (i) Discuss the continuity of $f(x)$ defined by :

$$f(x) = \begin{cases} 1 & \text{for } -\infty < x < 0 \\ 1 + \sin x & 0 \leq x < \frac{\pi}{2} \\ 2 + \left(x - \frac{\pi}{2}\right)^2 & \frac{\pi}{2} \leq x < \infty \end{cases}$$

- (ii) Find A^{-1} , by using Cayley-Hamilton theorem,

$$\text{where } A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

BCA-105(N)

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B. C. A. (First Semester)
EXAMINATION, Nov./Dec., 2018

(New Course)

Paper Fifth

MATHEMATICS—I

Time : Three Hours] [Maximum Marks : 75

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : Attempt all questions. Each question carries 3 marks.

1. (A) Using Cramer's rule, solve the following system of equations :

$$5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

- (B) Prove that :

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(C-95) P. T. O.

- (C) Discuss the discontinuity of the function :

$$f(x) = \frac{\frac{1}{e^x} - 1}{e^x + 1} \text{ for } x \neq 0 \text{ and } f(0) = 0.$$

- (D) Discuss the Lagrange's Mean Value theorem for

$$f(x) = x^3 \text{ in } -2 \leq x \leq 2 \text{ and find C.}$$

- (E) Find the maximum and minimum values of

$$x^5 - 5x^4 + 5x^3 - 10.$$

- (F) Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx$$

- (G) Prove that :

$$\beta(m+1, n) + \beta(m, n+1) = \beta(m, n) \quad m, n > 0$$

- (H) Find the value of λ so that the vectors :

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k},$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$$

are coplanar.

- (I) If $x = a(\theta + \sin \theta)$ and $y = a(1 + \cos \theta)$, then prove that $\frac{dy}{dx} = -\tan \frac{\theta}{2}$.

(C-95)

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

2. (i) Verify Rolle's theorem for the function

$$f(x) = x^3 - 4x \quad x \in (-2, 2).$$

- (ii) Find the value of a and b in order that

$$\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} \text{ may be equal to 2.}$$

3. Trace the curve $x^5 + y^5 = 5a^2x^2y$.

4. If $y = \sin(m \sin^{-1} x)$, prove that :

$$(1 - x^2)y_2 - xy_1 + m^2y = 0$$

and deduce that :

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

5. (i) Prove that :

$$\int_0^{\frac{\pi}{4}} \log_e (1 + \tan x) \, dx = \frac{\pi}{8} \log_e 2$$

- (ii) Evaluate :

$$\int \frac{x+4}{3+2x-x^2} \, dx$$

Section—C

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

6. (i) Show that the vectors $v_1 = \{1, 2, 4\}$, $v_2 = \{2, -1, 3\}$, $v_3 = \{0, 1, 2\}$, $v_4 = \{-3, 7, 2\}$ are linearly dependent and find the relation between them.

(C-95) P. T. O.